WEEKLY TEST TYJ -01 MATHEMATICS SOLUTION 08 SEPTEMBER 2019

31. (c) $=(1+3x)^2(1-2x)^{-1}$

$=(1+3x)^2\left(1+2x+\frac{1.2}{2.1}(-2x)^2+....\right)$

$=(1+6x+9x^2)(1+2x+4x^2+8x^3+....)$

Therefore coefficient of x^3 is $(8+24+18)=50$.

32. (c) $(1-x)^{3/2}$

$=\left[1+\frac{3}{2}(-x)+\frac{3}{2}.\frac{1}{2}.\frac{1}{2!}(-x)^2+\frac{3}{2}.\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{3!}(-x)^3+...\right]$

$=1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{x^3}{16}$ (only four terms).

33. (c) $(1+x)^{27/5}$

$T_{r+1}=\frac{n(n-1)(n-2)......(n-r+1)}{r!}(x)^r$

For first negative term $n-r+1<0$; $r>\frac{32}{5}$.

$\therefore$ First negative term is 8^{th} term.

34. (c) Given expression

$=2[x^5+{}^5C_2x^3\{(x^3-1)^{1/2}\}^2+{}^5C_4x\{(x^3-1)^{1/2}\}^4]$

$=2[x^5+10x^3(x^3-1)+5x(x^3-1)^2]$

$=5x^7+10x^6+x^5-10x^4-10x^3+5x$,

which is a polynomial of degree 7.

35. (b) $\frac{(n+1)(n+2)}{2}=45$ or $n^2+3n-88=0\Rightarrow n=8$..

36. (b) $(2+\sqrt{2})^4=(\sqrt{2})^4(\sqrt{2}+1)^4$

$=4[{}^4C_0+{}^4C_1(\sqrt{2})+{}^4C_2(\sqrt{2})^2+{}^4C_3(\sqrt{2})^3+{}^4C_4(\sqrt{2})^4]$

$=4\left[1+4\sqrt{2}+\frac{4.3}{2}.2+\frac{4.3.2}{1.2.3}.2\sqrt{2}+4\right]$

$=4[1+4\sqrt{2}+12+8\sqrt{2}+4]\ =\ 4[17+12\sqrt{2}]$

$=4[17+(\simeq 17)]\ =\ 4[34]=136$.

37. (a) We know that $n!$ terminates in 0 for $n\geq 5$ and 3^{4n} terminator in 1, ($\because\ 3^4=81$)

$\therefore 3^{180}=(3^4)^{45}$ terminates in 1

Also $3^3=27$ terminates in 7

$\therefore\ 3^{183}=3^{180}3^3$ terminates in 7.

$\therefore\ 183!+3^{183}$ terminates in 7

i.e. the digit in the unit place $=7$.

38. (b) $5^{99}=(5)(5^2)^{49}=5(25)^{49}=5(26-1)^{49}$

$=5\times(26)\times(\text{Positive terms})-5$, So when it is divided by 13 it gives the remainder – 5 or (13 – 5) *i.e.,* 8.

39. (b) $(x+a)^n + (x-a)^n = 2\,[x^n + {}^nC_2 x^{n-2}a^2 + {}^nC_4 x^{n-4}a^4 +$
${}^nC_6 x^{n-6}a^6 + ...]$

Here, $n = 6, x = \sqrt{2}, a = 1$; ${}^6C_2 = 15, {}^6C_4 = 15, {}^6C_6 = 1$

$\therefore (\sqrt{2}+1)^6 (\sqrt{2}-1)^6 = 2[(\sqrt{2})^6 + 15.(\sqrt{2})^4.1$
$+15(\sqrt{2})^2.1 + 1.1]$

$= 2[8 + 15\times 4 + 15\times 2 + 1] = 198$

40. (b) We have $(1+x^2)^5(1+x)^4$

$= ({}^5C_0 + {}^5C_1x^2 + {}^5C_2x^4 + ...)\,({}^4C_0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4)$

So coefficient of x^5 in $[(1+x^2)^5(1+x)^4]$

$= {}^5C_2.{}^4C_1 + {}^4C_3.{}^5C_1 = 60.$

41. (b) $(x-1)(x-2)(x-3)....(x-100)$

Number of terms = 100;

$\therefore$ Coefficient of x^{99} = $(x-1)(x-2)(x-3)...(x-100)$

$= (-1-2-3-......-100) = -(1+2+.....+100)$

$= -\frac{100\times 101}{2} = -5050.$

42. (a) In the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$, the general term is $T_{r+1} = {}^{11}C_r (ax^2)^{11-r}\left(\frac{1}{bx}\right)^r = {}^{11}C_r a^{11-r}\frac{1}{b^r}x^{22-3r}$

For x^7, we must have $22 - 3r = 7 \Rightarrow r = 5$, and the coefficient of $x^7 = {}^{11}C_5.a^{11-5}\frac{1}{b^5} = {}^{11}C_5\frac{a^6}{b^5}$

Similarly, in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$, the general term is $T_{r+1} = {}^{11}C_r(-1)^r\frac{a^{11-r}}{b^r}.x^{11-3r}$

For x^{-7} we must have, $11 - 3r = -7 \Rightarrow r = 6$, and the coefficient of x^{-7} is ${}^{11}C_6\frac{a^5}{b^6} = {}^{11}C_5\frac{a^5}{b^6}$.

As given, ${}^{11}C_5\frac{a^6}{b^5} = {}^{11}C_5\frac{a^5}{b^6} \Rightarrow ab = 1$.

43. (a) Let the coefficient of three consecutive terms *i.e.* $(r+1)^{th}$, $(r+2)^{th}$, $(r+3)^{th}$ in expansion of $(1+x)^n$ are 165,330 and 462 respectively then, coefficient of $(r+1)^{th}$ term $= {}^nC_r = 165$

Coefficient of $(r+2)^{th}$ term $= {}^nC_{r+1} = 330$ and

Coefficient of $(r+3)^{th}$ term $= {}^nC_{r+2} = 462$

$\therefore \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1} = 2$

or $n - r = 2(r+1)$ or $r = \frac{1}{3}(n-2)$

and $\frac{{}^nC_{r+2}}{{}^nC_{r+1}} = \frac{n-r-1}{r+2} = \frac{231}{165}$

or $165(n-r-1) = 231(r+2)$ or $165n - 627 = 396r$

or $165n - 627 = 396\times\frac{1}{3}\times(n-2)$

or $165n - 627 = 132(n-2)$ or $n = 11$.

44. (d) ${}^{18}C_{2r+3} = {}^{18}C_{r-3} \Rightarrow 2r + 3 + r - 3 = 18 \Rightarrow r = 6$

45. (c) The general term in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is $T_{r+1} = {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)x^{18-3r}$

......(i)

Now, the coefficient of the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ ......(ii)

$=$ Sum of the coefficient of the terms x^0, x^{-1} and x^{-3} in $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

For x^0 in (i) above, $18-3r=0 \Rightarrow r=6$. For x^{-1} in (i) above, there exists no value of r and hence no such term exists. For x^{-3} in (i), $18-3r=-3 \Rightarrow r=7$

$\therefore$ For term independent of x, in (ii) the coefficient

$$=1\times {}^9C_6(-1)^6\left(\frac{3}{2}\right)^{9-6}\left(\frac{1}{3}\right)^6+2\times {}^9C_7(-1)^7\left(\frac{3}{2}\right)^{9-7}\left(\frac{1}{3}\right)^7$$

$$=\frac{9.8.7}{1.2.3}.\frac{3^3}{2^3}.\frac{1}{3^6}+2\frac{9.8}{1.2}(-1)\frac{3^2}{2^2}.\frac{1}{3^7}=\frac{7}{18}-\frac{2}{27}=\frac{17}{54}.$$